

Elements stability: a microcosmos effect?

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The electronic structure of heavy elements, when described in a space-time which the metric is affected by the electromagnetic interaction, presents instabilities. These instabilities increase with the atomic number, and above a critical value, become important. The consistency of this theory [1],[2] and a formulation based on the energy-momentum tensor is also investigated. With this procedure, a dynamical cut-off appears in a natural way, and the field equations for the general quantum mechanics are determined.

I. INTRODUCTION

A fundamental question in the understanding of the Nature is if large-scale systems, such as stars or galaxies are submitted to the same principles that rules the microscopic world. Should these small systems be considered as microcosmos?

The microscopic world, is known to be explained in terms of the quantum mechanics and a key element for its development was the study of the atomic structure. The success of the quantum theory in studying these systems, confirmed the foundations of the theory and allowed the description of a large number of systems such as molecules and nuclei. The understanding of the physical reality by the quantum mechanics has been greatly improved with the Dirac theory [3], that formulated the quantum mechanics in the framework of the special relativity with remarkable success.

On the other hand, the Einstein general theory of relativity is another great achievement, that deals with the structure of the space-time, and when applied to very large systems, gives very precise results also. Since 1930, many works (as for example [15]-[17]) have been proposed with the objective of quantizing the gravity. Looking at these questions from a different point of view, a question that must remain in many heads is if the general relativity formulation may have effects, or at least, some analogy in the study of very small systems, such as atoms or elementary particles.

In practical terms, one aspect of this question is if the microscopic world interactions, the weak, electromagnetic and the strong, may affect significantly the space-time metric and if this proposition may have any observable effect. In this description, the gravitational forces may be neglected, and it is a good approximation, due to the small masses of the considered particles. In [1], these ideas have been formulated considering a particle in a region with a potential, that affects the metric, and the wave equations for spin-0 and spin-1/2 particles have been proposed, generating very interesting results.

The simplest systems where this theory could be tested are the one electron atoms, and the calculation of the deuterium spectrum has shown a clear numerical improvement when compared with the usual Dirac spectrum [3], [4] (also proposed by Sommerfeld [5]), with a percentual deviation from the experimental results approximately five times smaller, near one additional digit of precision.

An interesting fact that appeared from this theory, is the existence of horizons of events inside these quantum systems, with sizes that are not negligible. This propriety, that is related with the existence of a trapping surface at r_0 , as it was defined by Penrose [6], in [2] have been successfully used in order to describe quark confinement. Solving the quantum wave equations [2], quark confinement has been obtained, without the need of introducing confining potentials, as it is currently done [7]- [10]. The confinement obtained in this way is a strong confinement, as the quarks cannot reach r_0 .

In order to compliment this theory, the energy-momentum tensor $T^{\mu\nu}$ should be included in this formulation. This study is made in Sect. 2 and consistence with the previous results is found. In Sect. 3, the effect of instability in heavy elements is studied, and in Sect. 4, the conclusions are shown.

II. FIELD EQUATIONS AND METRIC

In quantum systems, the electromagnetic and strong interactions dominate and the gravitational interaction is negligible, as the masses of the considered particles are very small. Consequently, the gravitational potential may be

turned off, and then, the space-time metric is affected only by the other interactions.

In this section, the field equations for particles subjected to non gravitational interactions will be obtained. For this purpose a brief review of the results of [1] will be made, and then, this results will be related with a formulation based on the energy-momentum tensor.

For simplicity, a system with spherical symmetry will be considered, but the basic ideas can be generalized to systems with arbitrary metrics. If the spherical symmetry is considered, the space-time may be described by the metric derived in [1], that is very similar to the Schwarzschild metric [11],[12],

$$ds^2 = \xi d\tau^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2) - \xi^{-1} dr^2 , \quad (1)$$

where r , θ and ϕ are the particle coordinates, $\xi(r)$ is determined by the interaction potential $V(r)$, and is a function only of r , for a time independent interaction. In this case, the metric tensor $g_{\mu\nu}$ is diagonal

$$g_{\mu\nu} = \begin{pmatrix} \xi & 0 & 0 & 0 \\ 0 & -\xi^{-1} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2 \theta \end{pmatrix} . \quad (2)$$

The energy relation for this system is [1]

$$\frac{E}{\sqrt{\xi}} = \sqrt{p^2 c^2 + m_0^2 c^4} , \quad (3)$$

therefore,

$$E(\vec{\beta} = 0) = E_0 \xi^{1/2} = E_0 + V , \quad (4)$$

where $\vec{\beta}$ is the particle velocity. This relation means that in the rest frame of the particle, the energy is simply due to the sum of its rest mass E_0 with the potential, and then,

$$\xi^{1/2} = 1 + \frac{V}{mc^2} . \quad (5)$$

Applying these ideas in the study of one electron atoms, V is the Coulomb potential

$$V(r) = -\frac{\alpha Z}{r} , \quad (6)$$

α is the fine structure constant and Z is the atomic number. Consequently, the function ξ is given by

$$\xi = 1 - \frac{2\alpha Z}{mc^2 r} + \frac{\alpha^2 Z^2}{m^2 c^4 r^2} , \quad (7)$$

where m is the electron mass. These expressions determine the horizon of events at r_0 , that appears from the metric singularity $\xi(r_0)=0$, and using the values of [13], one finds

$$r_0 = \frac{\alpha Z}{mc^2} = 2.818 Z \text{ fm} , \quad (8)$$

that is not a negligible value at the atomic scale.

Now, let us turn our attention to a description based on the energy-momentum tensor. If one consider a field generated by the electromagnetic interaction, the energy-momentum tensor is

$$T^{\mu\nu} = \epsilon_0 \left(F^{\mu\alpha} F_{\alpha}^{\nu} - \frac{1}{4} \delta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right) , \quad (9)$$

and it is related to the space-time geometry by the field equations

$$R_{\nu}^{\mu} - \frac{1}{2} R \delta_{\nu}^{\mu} = -A T_{\nu}^{\mu} , \quad (10)$$

where A is a constant to be determined.

In an atom, $T^{\mu\nu}$ is determined by the nuclear electrostatic field, and the nonvanishing components are

$$T_0^0 = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) , \quad (11)$$

and $T_r^r = -T_0^0$. For a central electrostatic field with charge Ze , T_0^0 is just

$$T_0^0 = \frac{\epsilon_0 E^2}{2} = \frac{Z^2 \alpha}{8\pi r^4} . \quad (12)$$

Eq. (10) may be used to determine the ξ function observing that in the given metric

$$\begin{aligned} \frac{e^{-B}}{r^2} \left(r \frac{dB}{dr} - 1 \right) + \frac{1}{r^2} &= AT_0^0 \\ \frac{e^{-B}}{r^2} \left(r \frac{dB}{dr} + 1 \right) - \frac{1}{r^2} &= -AT_0^0 , \end{aligned} \quad (13)$$

that have the solution [14]

$$\xi(r) = e^{-B} = 1 - \frac{c^2 AM(r)}{4\pi r} \quad (14)$$

with

$$M(r) = \int_0^r \frac{4\pi}{c^2} (r')^2 T_0^0 dr' . \quad (15)$$

If the particle is outside the horizon of events, that is the case of the electron in an atom, it will be affected just by the part of the field located in the region external to the horizon of events, and the integration (15) must be performed in the interval $r_0 \leq r' \leq r$,

$$M(r) = m_0 + \int_{r_0}^r \frac{4\pi}{c^2} (r')^2 T_0^0 dr' = m_0 + \frac{Z^2 \alpha}{2 c^2} \left(\frac{1}{r_0} - \frac{1}{r} \right) , \quad (16)$$

where m_0 is a constant of integration. So,

$$M(r) = m_0 + \frac{mZ}{2} - \frac{Z^2 \alpha}{2 c^2 r} \quad (17)$$

$$M(r \rightarrow \infty) = m_0 + \frac{mZ}{2} \quad (18)$$

and

$$\xi = 1 - \frac{c^2 AZ}{4\pi r} \left[m_0 + \frac{mZ}{2} \right] + \frac{AZ^2 \alpha}{8\pi r^2} \quad (19)$$

that is solution of (13). The constants A and m_0 may be obtained now, comparing the expression (19), that is determined by the energy-momentum tensor and the field equations, with (7), determined by the energy relation (3).

Identifying the terms r^{-1} and r^{-2} we find

$$\begin{aligned} A &= \frac{8\pi \alpha}{m^2 c^4} \\ m_0 &= \frac{mZ}{2} . \end{aligned} \quad (20)$$

The field equation is then

$$R_\nu^\mu - \frac{1}{2} R \delta_\nu^\mu = -\frac{8\pi \alpha}{m^2 c^2} T_\nu^\mu . \quad (21)$$

So, in this system, due to the interaction, the electron energy decrease with r , until it reaches the value $E(r_0) = 0$ at the horizon of events. At large distances, $E(\infty) = m$, the energy is obviously due only to its rest mass.

Considering that the strong interactions may be approximated by a strong Coulomb field [2], with $\alpha \sim 1$, one may use the field equation (21) in order to study strongly interacting systems. But if one wants to describe the strong interactions by the Yang-Mills field, (21) is determined by the correct coupling constant α , and an Yang-Mills energy-momentum tensor.

III. Z LIMIT

The quantum wave equation for spin-1/2 particles, in the metric (1) is [1]

$$\frac{i\hbar}{\xi} \frac{\partial}{\partial t} \Psi = \left(-i\hbar c \vec{\alpha} \cdot \vec{\nabla} + \beta m_0 c^2 \right) \Psi \quad (22)$$

where Ψ is a four-component spinor. In a first view, this equation may seem very similar to the Dirac equation, but it shows some important differences. The first one, is the numerical aspect, applying this theory to the deuterium atom [1], the accord with the experimental values of the energy levels is improved in comparison with the Dirac spectrum and shows deviations from the data of the order of 0.005%. Another characteristic of the general quantum mechanics, is the existence of the horizon of events at r_0 . In the Hydrogen atom, this fact is not important, $r_0 \sim 2.8$ fm, and, from the solution of (22), an electron with energy of the order of few eV has a very small probability of being found in this region, so, in practical terms, no effect is observed. However, for heavy elements, that present larger nuclear charges, the value of r_0 may not be negligible as it increases with Z . For example, for the mercury, $Z=80$, so, $r_0=224$ fm, should this value be important? In this section, the main objective is to study this effect.

An estimate of the spacial region where the $1S_{1/2}$ electron has significant probability of being found is the interval $r_{min} = \langle r \rangle - \Delta r \leq r \leq \langle r \rangle + \Delta r$, where $\langle r \rangle$ is the expectation value of r and $\Delta r = \sqrt{\langle r^2 \rangle - \langle r \rangle^2}$ is the statistical fluctuation of r around $\langle r \rangle$. From the solution of eq. (22) [1], [2], one may determine theses quantities.

Solving eq. (22), the spacial part of Ψ , that is a four component spinor, may be written as

$$\psi = \begin{pmatrix} F(r) \chi_{\kappa}^{\mu} \\ iG(r) \chi_{-\kappa}^{\mu} \end{pmatrix}, \quad (23)$$

with the aid of the usual two component spinors, χ_{κ}^{μ} [1], where

$$\begin{aligned} \kappa &= l & \text{for } j &= l - 1/2, \\ \kappa &= -l - 1 & \text{for } j &= l + 1/2. \end{aligned} \quad (24)$$

For the $1S_{1/2}$ electrons, the quantum numbers are $n = 1$, $l = 0$ and $\kappa = -1$, and a good approximation for the ground state eigenfunctions is

$$\begin{aligned} F_{1,-1} &= N \sqrt{m(1+s)} r^{s-1} e^{-\gamma m r} \\ G_{1,-1} &= -N \sqrt{m(1-s)} r^{s-1} e^{-\gamma m r}, \end{aligned} \quad (25)$$

where

$$\gamma = Z e^2 \quad (26)$$

and

$$s = \sqrt{1 - \gamma^2}. \quad (27)$$

The normalization is made using

$$\begin{aligned} \int \psi^* \psi d^3x &= \int_0^\infty (|F|^2 + |G|^2) r^2 dr = \\ &= 2N^2 m^2 \int_0^\infty r^{2s} e^{-2\gamma m r} dr = 1, \end{aligned} \quad (28)$$

that determines the constant

$$N = \frac{1}{m} \frac{(2\gamma m)^{s+1/2}}{\sqrt{2\Gamma(2s+1)}}. \quad (29)$$

So,

$$\begin{aligned} \langle r \rangle &= 2N^2 m^2 \int_0^\infty r^{2s+1} e^{-2\gamma m r} dr = \frac{2s+1}{2\gamma m} \\ \langle r^2 \rangle &= 2N^2 m^2 \int_0^\infty r^{2s+2} e^{-2\gamma m r} dr = \frac{(2s+2)(2s+1)}{(2\gamma m)^2}, \end{aligned} \quad (30)$$

and from these results,

$$\Delta r = \frac{\sqrt{2s+1}}{2\gamma m} \quad (31)$$

that gives

$$r_{min}(Z) = \frac{2.6459226 \cdot 10^4}{Z} \left(2\sqrt{1 - \alpha^2 Z^2} + 1 - \sqrt{2\sqrt{1 - \alpha^2 Z^2}} \right) \text{ fm} . \quad (32)$$

Another value of interest, is the value of r where the wave function reaches the maximum, that may be calculated using

$$\frac{d(rF)}{dr} = 0 = \frac{\sqrt{1 - \alpha^2 Z^2}}{r} - \alpha m Z , \quad (33)$$

that gives

$$r_{peak} = \frac{\sqrt{1 - \alpha^2 Z^2}}{\alpha m Z} , \quad (34)$$

and from G , one gets the same relation.

The results for r_0 , r_{min} and r_{peak} are shown in Fig. 1-3.

In this framework, atomic instability effects are expected to occur in elements that present $r_{min} < r_0$, what means that the $1S_{1/2}$ electron has significant probability of reaching the horizon of events. In this case, the ground state is unstable, and from (32), one notes that it happens for $Z > 92.39$. In view of these results (see Fig. 1) there is no surprise in the fact that elements with atomic numbers greater than the critical value $Z=92$ (Uranium) are not found in Nature, and exists only if synthesized artificially by man. If $r_0 = r_{peak}$, the element will be very unstable, and it happens for

$$Z \sim \frac{0.618}{\alpha} \sim 107.73 \quad (35)$$

that is a little bit above the last element, Rf (Rutherfordium), that has $Z=104$, fact that shows one more time that the theory is in agreement with the observable world.

A point that must be remarked, is that these results are obtained only studying the electronic structure of the elements, without considering their nuclear structures. As it is well known, some aspects of the interaction responsible for the shell structure of the nuclei need to be understood, and one factor that may be considered, specially for heavy elements, is the effect of instability presented in this work.

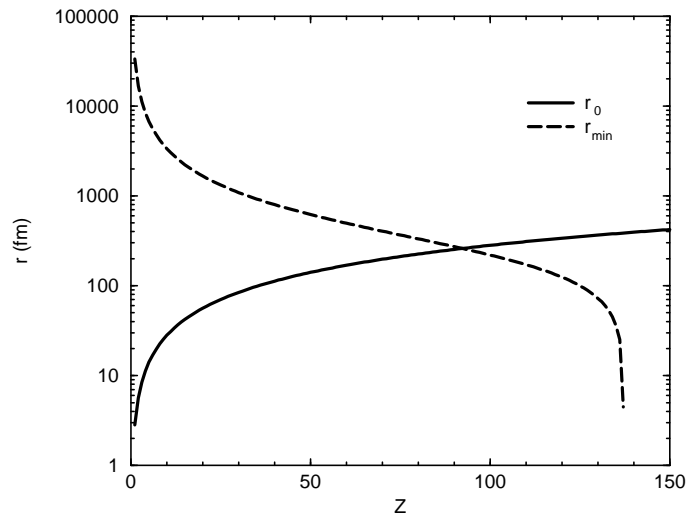


FIG. 1: Comparison between r_0 and r_{min} as functions of the nuclear charge, Z .

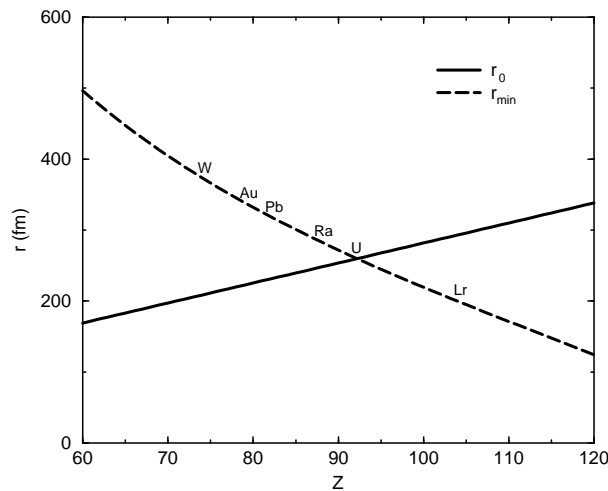


FIG. 2: Comparison between r_0 and r_{min} .

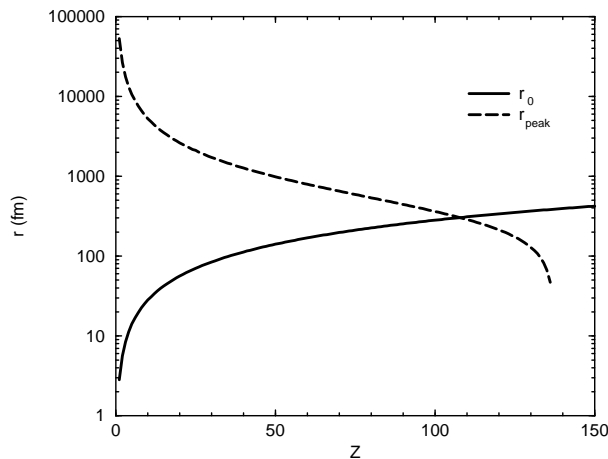


FIG. 3: Comparison between r_0 and r_{peak} as functions of the nuclear charge, Z .

IV. SUMMARY AND CONCLUSIONS

In this paper the study of the general quantum mechanics is been continued. The metric is determined by the interaction of quantum objects, such as electrons and quarks. The effect of the energy-momentum tensor of electromagnetic and strong interactions in the space-time has been considered, and with this procedure the the constant of the field equations has been calculated. The results obtained are consistent with the ones of [1].

One can observe the presence of the mass in the constant A , what does not happen in the general relativity, fact that is due to the equivalence principle. Observing that $A \propto g/m^2$, one concludes that the effect of the curvature of space-time, for a particle of mass m in a field, decreases for large masses and increases for small masses and for large coupling constants. It is interesting to note that a dynamical cut-off is determined by this theory, as it was used in eq. (16), providing correct results.

Another observation that must be made is that initially, spherical symmetry has been supposed, but eq. (21) is general, independent of the symmetry of the system. This equation may also be used with the inclusion of other interactions, as the Yang-Mills one, for example, considering the Yang-Mills field tensor $F_{\mu\nu}^a$ in the construction of the energy-momentum tensor, and quark confinement, from the results of this theory, is expected to occur.

One must remark that with the development of the general quantum mechanics, we are being able to explain many characteristics of the studied physical systems [1], [2], using the new proprieties that appears from the understanding of the geometry of the space-time. The instability effect presented in this paper is another result of the theory that does not appear in the usual quantum mechanics, despite the fact that it is an observable effect.

The Dirac theory introduced the special relativity in quantum mechanics, so it is very reasonable to think that the next step is to formulate the quantum mechanics in a way analogous of the one that the general relativity is. But in fact, the step proposed here, lead to a theory that absolutely is not a quantized version of general relativity, many differences occur and some concepts of the general relativity are not the same ones in this formulation. The atomic spectrum obtained in this way shows that the corrections of the energy levels, due to this general formulation of quantum mechanics with the inclusion of the electric interaction in the space-time metric, provide a quite impressive agreement with the experimental data, and is a strong evidence in the validation the theory. Another interesting result is the hadron model [2], where quark confinement is shown to be a consequence of this theory.

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